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Bergshoeff, E.A.

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# Kaluza-Klein Monopoles and Gauged Sigma Models

E.A. Bergshoeff<sup>a\*</sup>

<sup>a</sup>Institute for Theoretical Physics,  
 Nijenborgh 4, 9747 AG Groningen, The Netherlands

We review some aspects of branes. In particular, we discuss the worldvolume theory describing the dynamics of the Kaluza-Klein monopole which turns out to be a gauged sigma model. We also briefly review some recent applications of gauged sigma models to the worldvolume description of massive branes, i.e. branes moving in a background with a nonzero cosmological constant.

## 1. Introduction

Strings are one-dimensional spacelike structures that generalize the notion of a particle. The different vibration modes of the string correspond to (massless as well as massive) elementary particles. It is natural to extend this idea and also consider membranes which are two-dimensional extended structures. In that case the different vibration modes of the membrane correspond to elementary particles. Similarly, one may consider  $p$ -dimensional extended objects, or simply  $p$ -branes.

The following three aspects are relevant when discussing extended objects:

1. Worldvolume actions
2. Target space effective actions
3. Extended object solutions

These three aspects are related in different ways. For instance, for strings, i.e.  $p=1$ , the starting point is the worldsheet 1-brane action which in this case is a Nambu-Goto type action for the embedding coordinates  $X^\mu$ . There exist several techniques to relate this world-sheet action (in a curved background) to a target space effective action which, at low energies and weak coupling, is given by a supergravity theory. This target space action allows different kinds of  $p$ -brane solutions,

i.e. solutions with  $p$  spacelike isometries (for a review of extended object solutions, see [1]). In particular, it allows a 1-brane solution. There are two ways to relate this solution to the original worldsheet 1-brane action. First of all, the effective dynamics of the 1-brane is described by the original Nambu-Goto action. Secondly, the same Nambu-Goto action also occurs as a source term in the 1-brane solution. It should be noted that one extended object solution may lead to different worldvolume actions. This happens whenever the charge of the solution is carried by the Neveu-Schwarz/Neveu-Schwarz (NS/NS) 2-form or its dual. In that case there are three possibilities. One may embed the solution in the NS/NS, IIA or IIB sector. This happens for instance for the above-mentioned 1-brane solution which is often called the fundamental string or P1-solution [2]:

$$P1 \rightarrow \begin{cases} \text{NS}, \\ \text{IIA}, \\ \text{IIB}. \end{cases} \quad (1)$$

In general, the target space effective action always contains, among many other terms (including terms with fermions), a metric  $g_{\mu\nu}$ , a dilaton  $\phi$  and a  $p+1$ -form gauge field  $A_{(p+1)}$  with curvature  $F_{(p+2)}$ . In the Einstein frame the part of the action containing these three fields is given in terms of three parameters, the target space dimension  $d$ , the order  $p+1$  of the gauge field form and a dilaton coupling parameter  $\alpha$ :

$$\mathcal{L}_{E,d} = \sqrt{|g|} \left[ R + \frac{1}{2}(\partial\phi)^2 \right]$$

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$$+ \frac{(-)^{p+1}}{2(p+2)!} e^{a\phi} F_{(p+2)}^2 \Big]. \quad (2)$$

When considering extended object solutions of the above actions it is natural to first consider the following class of so-called “two-block solutions”:

$$\begin{aligned} ds^2 &= H^\alpha dx_{(p+1)}^2 - H^\beta dx_{(d-p-1)}^2, \\ e^{2\phi} &= H^\gamma, \\ F_{0\dots pI} &= \delta\partial_I H^\epsilon, \end{aligned} \quad (3)$$

with  $H = H(x^I)$  a harmonic function of  $x^I$  ( $I = 1, \dots, d-p-1$ ) and  $\alpha, \dots, \epsilon$  constant parameters. Note that the metric is naturally split into two blocks: the first  $(p+1)$  directions in the metric correspond to the worldvolume of the extended object solution while the last  $(d-p-1)$  directions are transverse to the extended object.

It turns out that for  $d > 2$ , given the above action and Ansatz, there is a unique solution given by<sup>2</sup>

$$\begin{aligned} \alpha &= -\frac{4(d-p-3)}{(d-2)\Delta}, \\ \beta &= \frac{4(p+1)}{(d-2)\Delta}, \\ \gamma &= \frac{4a}{\Delta}, \\ \delta^2 &= \frac{4}{\Delta}, \quad \epsilon = -1, \end{aligned} \quad (4)$$

with

$$\Delta = a^2 + 2\frac{(p+1)(d-p-3)}{d-2}. \quad (5)$$

One may divide the brane solutions of string theory into three types.

### 1. Elementary $p$ -branes

Elementary  $p$ -branes are described by a Nambu-Goto worldvolume action that has no dilaton coupling in front of it:

$$S_{\text{NG}} \sim \sqrt{|g|}, \quad (6)$$

showing that the mass  $M$  of the object is independent of the string coupling constant  $g = e^{<\phi>}$ .

<sup>2</sup>We use here a form of the solution as given in [3].

An example is the fundamental string or P1-solution [2]. Taking in the above action  $(d, p, a) = (10, 1, -2)$  one finds that the P1-solution in string frame is given by:

$$P1 \left\{ \begin{aligned} ds_{S,10}^2 &= H^{-1} dx_{(2)}^2 - dx_{(8)}^2, \\ e^{2\phi} &= H^{-1}, \\ F_{01I} &= \partial_I H^{-1}. \end{aligned} \right. \quad (7)$$

### 2. Solitonic $p$ -branes

Solitonic  $p$ -branes are described by a Nambu-Goto worldvolume action that has a  $e^{-2\phi}$  dilaton coupling in front of it:

$$S_{\text{NG}} \sim e^{-2\phi} \sqrt{|g|}, \quad (8)$$

showing that the mass  $M$  of the object is proportional to the inverse squared of the string coupling constant:  $M \sim 1/g^2$ .

An example is the solitonic five-brane or P5-solution [4,5]. Taking  $(d, p, a) = (10, 5, 2)$  one finds that the P5-solution is given by

$$P5 \left\{ \begin{aligned} ds_{S,10}^2 &= dx_{(6)}^2 - H dx_{(4)}^2, \\ e^{2\phi} &= H, \\ F_{012345I} &= \partial_I H^{-1}. \end{aligned} \right. \quad (9)$$

### 3. Dirichlet branes [6]

D $p$ -branes ( $0 \leq p \leq 9$ ) are described by a so-called Dirac-Born-Infeld worldvolume action which contains the Nambu-Goto action upon setting the Born-Infeld 1-form equal to zero and which has a  $e^{-\phi}$  dilaton coupling in front of it:

$$S_{\text{NG}} \sim e^{-\phi} \sqrt{|g|}, \quad (10)$$

showing that the mass  $M$  of the object is proportional to the inverse of the string coupling constant:  $M \sim 1/g$ .

A special feature of the D $p$ -brane solutions is that their charge is carried by a Ramond-Ramond (R-R) gauge field. In ten-dimensional IIA and IIB supergravity the following potentials occur:

$$\text{IIA} : A_{(1)}, A_{(3)}, A_{(5)}, A_{(7)}, A_{(9)}, \quad (11)$$

$$\text{IIB} : A_{(0)}, A_{(2)}, A_{(4)}^+, A_{(6)}, A_{(8)}. \quad (12)$$

Note that the 5-form curvature of the 4-form potential is self-dual. The 9-form potential in

the IIA theory describes a cosmological constant. The above potentials lead to the following Dp-brane solutions ( $0 \leq p \leq 9$ ):

$$Dp \begin{cases} ds_{S,10}^2 &= H^{-\frac{1}{2}} dx_{(p+1)}^2 - H^{\frac{1}{2}} dx_{(9-p)}^2, \\ e^{2\phi} &= H^{-1/2(p-3)}, \\ F_{0\dots pI} &= \partial_I H^{-1}, \end{cases} \quad (13)$$

with  $H = H(x^I)$ . The extreme cases  $p = -1$  and  $p = 9$  are special. The D9-solution describes flat Minkowski space (the open superstring can move anywhere in the worldvolume of this 9-brane). The D-1-solution, or D-instanton corresponds to a flat Euclidean space with a non-zero dilaton and  $A_{(0)}$  [7].

All solutions of Type IIA superstring theory have an eleven dimensional interpretation [8] (except for the D8-brane). Due to its relation with M-theory these  $d=11$  solutions are called M-branes. The eleven-dimensional Lagrangian is given by

$$\begin{aligned} \mathcal{L}(d=11) &= \sqrt{|g|} \left( R - \frac{1}{2 \cdot 4!} F_{(4)}^2 \right) \\ &- \frac{1}{(3! \cdot 4!)^2} \epsilon A_{(3)} F_{(4)} F_{(4)}. \end{aligned} \quad (14)$$

Indeed, the fundamental string (P1) [2] and the solitonic five-brane (P5) [4,5] are the double dimensional reduction of the eleven dimensional M2-brane [9]

$$\begin{aligned} ds_{E,11}^2 &= H^{-2/3} dx_{(3)}^2 - H^{1/3} dx_{(8)}^2, \\ F_{012I} &= \partial_I H^{-1}, \end{aligned} \quad (15)$$

and the direct dimensional reduction of the eleven dimensional M5-brane [10]

$$\begin{aligned} ds_{E,11}^2 &= H^{-1/3} dx_{(6)}^2 - H^{2/3} dx_{(5)}^2, \\ F_{012345I} &= \partial_I H^{-1}, \end{aligned} \quad (16)$$

respectively. The Dirichlet D2- and D4-branes can be obtained from the M2-brane and M5-brane via direct and double dimensional reduction, respectively. The D0- and D6-branes in the IIA theory are related to the purely gravitational Brinkmann wave [11] (W) and the Kaluza-Klein monopole [12] (KK) in eleven dimensions. These eleven

dimensional solutions also have their counterparts in  $D = 10$ , which we denote by W and KK. Each of these solutions preserves 1/2 of the  $D = 11$  (or  $D = 10, N = 2$ ) supersymmetry. In Figure 1 (taken from [13]) we summarize the relationship between these  $d = 10$  IIA and  $d = 11$  solutions.

The eleven dimensional interpretation of the D8-brane [14,15] is still a mystery. Presumably, it is related to a 9-brane<sup>3</sup> in  $d = 11$ . The direct reduction of such a 9-brane is expected to lead to  $D = 10$  Minkowski space.

From Figure 1, we see that in order to relate the branes of string theory to those of M-theory we need to extend the class of M-brane and D-brane solutions to include waves and monopoles and in  $d=10$  we must include the P1- and P5-solutions:

$$\begin{aligned} \text{M-branes} &\rightarrow \text{M-branes} \\ &+ \text{waves} + \text{monopoles}, \end{aligned} \quad (17)$$

$$\begin{aligned} \text{D-branes} &\rightarrow \text{D-branes} \\ &+ \text{waves} + \text{monopoles}, \\ &+ \text{P1} + \text{P5}. \end{aligned} \quad (18)$$

We first discuss the the wave solution. The Brinkmann wave in  $d$  dimensions is given by the metric [11]

$$ds^2 = 2du dv + 2K(u, \vec{x}) du^2 - d\vec{x}^2, \quad (19)$$

with  $d\vec{x}^2 = dx_2^2 + \dots + dx_{(d-1)}^2$  and where we have used light-cone coordinates

$$u = \frac{1}{\sqrt{2}}(t + z), \quad v = \frac{1}{\sqrt{2}}(t - z). \quad (20)$$

The function  $K(u, \vec{x})$  is harmonic in the variables  $t + z, x_2, \dots, x_{(d-1)}$ . In ten dimensions the wave solution is  $T$ -dual to the P1-solution.

A special example of a Brinkmann wave is the gravitational shock wave

$$K(u, \vec{x}) = \delta(u) H(\vec{x}), \quad (21)$$

where the function  $H(\vec{x})$  is a harmonic function in  $\vec{x}$ . It turns out that the sigma model action

<sup>3</sup>The conjectured 9-brane is also discussed in [15–20].

corresponding to the Brinkmann wave is given by the action of a massless particle

$$S = \int d\tau \frac{1}{e} \dot{X}^\mu \dot{X}^\nu g_{\mu\nu}(X). \quad (22)$$

More precisely, a massless particle moving in the  $z$ -direction is the source of a gravitational shock wave. The embedding coordinates of such a massless particle are given by

$$U(\tau) = 0, V(\tau) = \sqrt{2}\tau, \vec{X}(\tau) = 0. \quad (23)$$

One may verify that indeed the gravitational wave, together with the above particle configuration, is a solution to the equations of motion that follow from the target space effective action to which the massless particle action is added as a source term [21].

The situation of the Kaluza-Klein monopole is less clear. In fact, up to recently, not much was known about its worldvolume structure. In the next section we will discuss some recent progress in this direction.

## 2. The Kaluza-Klein Monopole

The contents of this section is based upon the work described in [22]<sup>4</sup>. Our starting point is the 11-dimensional Kaluza-Klein monopole or KK11 solution [12]<sup>5</sup>

$$ds^2 = \eta_{ij} dy^i dy^j - H^{-1} (dz + A_m dx^m)^2 - H (dx^m)^2, \quad (24)$$

where  $i, j = 0, \dots, 6$ ,  $m, n = 7, 8, 9$ , and  $z = x^{10}$  and where

$$F_{mn} = 2\partial_{[m} A_{n]} = \epsilon_{mnp} \partial_p H, \quad \partial_m \partial_m H = 0. \quad (25)$$

The solution (24) has 8 isometries and therefore it represents an extended object. At first sight one might think that the solution represents a 7-brane (with non-isotropic worldvolume directions) but it turns out that the isometry in the

direction  $z$  is special and cannot be interpreted as a worldvolume direction [20]. We are therefore dealing with a 6-brane, with a 7-dimensional worldvolume, that has an additional isometry in one of the 4 transverse directions.

The KK11 solution preserves half of the supersymmetry and must correspond, after gauge fixing, to a 7-dimensional supersymmetric field theory. The natural candidate for such a field theory involves a vector multiplet with one vector and 3 scalars [20]. The vector corresponds to a Born-Infeld 1-form. We are now faced with a dilemma. Since the KK11-monopole moves in 11 dimensions, we have 11 embedding coordinates. Fixing the diffeomorphisms of the 7-dimensional worldvolume we are left with 4 instead of 3 scalars. These 4 scalars do not fit into a 7-dimensional vector multiplet. At this point one might argue that, to eliminate the extra scalar d.o.f., we need an extra diffeomorphism, i.e. an 8-dimensional worldvolume, but this would upset the counting of the worldvolume vector components. We therefore need a new mechanism to eliminate the unphysical scalar degree of freedom. It turns out that this can be done by gauging an Abelian isometry in the effective sigma-model, i.e. we propose to work with a *gauged sigma-model*.

A interesting feature of our proposal is that the “KK11-brane” couples to a scalar  $k$  constructed from the Killing vector  $k^\mu$  that generates the isometry we are gauging. The coupling manifests itself as a factor  $k^2$  in front of the kinetic term in the effective action. Since, in coordinates adapted to the isometry,  $g_{zz} = -k^2$  and the length of the  $z$ -dimension is

$$2\pi R_z = \int dz |g_{zz}|^{1/2} = \int dz k, \quad (26)$$

the tension of the KK11-brane is proportional to  $R_z^2$ .

To be concrete, we propose the following expression for the kinetic term of the KK11 effective action:

$$S_{\text{KK11}} = -T_{\text{KK11}} \int d^7 \xi k^2 \sqrt{|\det (\partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu} + k^{-1} \mathcal{F}_{ij})|}, \quad (27)$$

<sup>4</sup>For other recent work on the KK monopole, see e.g. [20, 23]

<sup>5</sup>The discussion below can easily be extended to arbitrary dimensions  $d$ . For simplicity we restrict ourselves to  $d=11$ .

where  $k^\mu$  is the Killing vector associated to the isometry direction  $z$  and

$$k^2 = -k^\mu k^\nu g_{\mu\nu}. \quad (28)$$

Furthermore,

$$\begin{aligned} \Pi_{\mu\nu} &= g_{\mu\nu} + k^{-2} k_\mu k_\nu, \\ \mathcal{F}_{ij} &= \partial_i V_j - \partial_j V_i - k^\mu \partial_i X^\nu \partial_j X^\rho C_{\mu\nu\rho}^{(3)}. \end{aligned} \quad (29)$$

The field  $C^{(3)}$  is the 3-form potential of 11-dimensional supergravity.

Observe that the components of the “metric”  $\Pi_{\mu\nu}$  in the directions of  $k^\mu$  vanish:

$$k^\mu \Pi_{\mu\nu} = 0. \quad (30)$$

$\Pi_{\mu\nu}$  is effectively a 10-dimensional metric and, in coordinates adapted to the isometry generated by  $k^\mu$ , the coordinate  $z$  and the corresponding field  $Z(\xi)$  associated to this isometry simply do not occur in the action.

Sometimes, it is convenient to consider only the purely gravitational part of the KK11 action, i.e. the part that one obtains after setting to zero the worldvolume vector field strength  $\mathcal{F}_{ij}$  ignoring the WZ term:

$$S_{\text{KK11}}^{\text{grav.}} = -T_{\text{KK11}} \int d^7 \xi k^2 \sqrt{|\det (\partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu})|}. \quad (31)$$

This part of the action can be written using an auxiliary worldvolume metric  $\gamma_{ij}$  in the Howe-Tucker form

$$S_{\text{KK11}}^{\text{grav.}} = -\frac{T_{\text{KK11}}}{2} \int d^7 \xi \sqrt{|\gamma|} \left[ k^{4/7} \gamma^{ij} \partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu} - 5 \right]. \quad (32)$$

Eliminating  $\gamma_{ij}$  from (32) leads to the expression given in (31). An alternative form of (32), which makes the relation with a gauged sigma-model clear, is obtained using an auxiliary worldvolume vector field  $C_i$ :

$$S_{\text{KK11}}^{\text{grav.}} = -\frac{T_{\text{KK11}}}{2} \int d^7 \xi \sqrt{|\gamma|} \left[ k^{4/7} \gamma^{ij} D_i X^\mu D_j X^\nu g_{\mu\nu} - 5 \right], \quad (33)$$

with the covariant derivative defined as

$$D_i X^\mu = \partial_i X^\mu + C_i k^\mu. \quad (34)$$

In the next section we will present a piece of evidence in favour of the proposed KK11 action by considering  $T$ -duality. More explicitly, we will show that the 10-dimensional heterotic Kaluza-Klein monopole (KKh) is  $T$ -dual to the d=10 solitonic five-brane (P5h)<sup>6</sup>. Note that the KK11 action of M-theory and the different actions of the 10-dimensional IIA/IIB theories are related via dimensional reduction. The heterotic KK monopole action is obtained by truncating the IIA/IIB actions. The worldvolume fields of the different actions that are related to the KK monopole via reduction and/or duality are given in Table 1.

### 3. $T$ -duality

We first wish to comment on the Buscher’s  $T$ -duality rules [26]. The standard derivation of the Buscher’s rules goes via a worldsheet duality transformation on the isometry scalar in the string effective action. This derivation only applies to strings but not to five-branes since the dual of a scalar is a scalar only in two dimensions. However, it turns out that there is an alternative way of deriving the Buscher’s rules which is more suitable for our purposes. Combining the fact that a wave is  $T$ -dual to a string and that the corresponding source terms are given by a massless particle and a string, respectively, one can show that the massless particle is  $T$ -dual to the string via reduction to d=9 dimensions [21]. This way of formulating Buscher’s  $T$ -duality is identical to the way the type II  $T$  duality between the  $R-R$  fields is treated [27]<sup>7</sup>. It is in this sense that we

<sup>6</sup>The  $T$ -duality between KK monopole and five-brane solutions corresponding to the effective action has been considered in the context of the magnetic ciral null model [24] and  $p$ -brane bound states [25].

<sup>7</sup>An interesting feature of this alternative derivation of Buscher’s rules is that the duality rule of the dilaton is needed already at the classical level.

Object	Worldvolume	Fields	# of d.o.f.
KK11	6+1	$X^\mu$	$11 - 7 - 1 = 3$
		$V_i$	$7 - 2 = 5$
D6	6+1	$X^\mu$	$10 - 7 = 3$
		$V_i$	$7 - 2 = 5$
KK10A	5+1	$X^\mu$	$10 - 6 - 1 = 3$
		$V_i$	$6 - 2 = 4$
		$S$	1
P5B	5+1	$X^\mu$	$10 - 6 = 4$
		$W_{ijk}$	4
M5	5+1	$X^\mu$	$11 - 6 = 5$
		$V_{ij}^+$	3
P5A	5+1	$X^\mu$	$10 - 6 = 4$
		$V_{ij}^+$	3
		$S$	1
KK10B	5+1	$X^\mu$	$10 - 6 - 1 = 3$
		$V_{ij}^+$	3
		$S$	1
		$T$	1

Table 1

The table gives the worldvolume fields and number of degrees of freedom of the different objects that are related to the d=11 Kaluza-Klein monopole via reduction and/or T-duality. The “-1” in the fourth column indicates that a scalar degree of freedom is eliminated by gauging an Abelian isometry.

show below that the KKh and P5h actions are T-dual to each other.

Our starting point is the 10-dimensional heterotic five-brane (P5h) action given by

$$S_{\text{P5h}} = -T_{\text{P5h}} \int d^6 \xi e^{-2\hat{\phi}} \sqrt{|\det(\partial_i \hat{X}^\mu \partial_j \hat{X}^\nu \hat{g}_{\mu\nu})|} + \text{WZ}. \quad (35)$$

A direct dimensional reduction of the P5h action leads to an action involving an extra worldsheet scalar  $S$ :

$$S_{\text{P5h}} = -T_{\text{P5h}} \int d^6 \xi e^{-2\phi} k^{-1} \sqrt{|\det(\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} - k^2 F_i F_j)|} + \text{WZ}, \quad (36)$$

with

$$F_i = \partial_i S - A_i. \quad (37)$$

On the other hand, the heterotic KK monopole action KKh is given by

$$S_{\text{KKh}} = -T_{\text{KKh}} \int d^6 \xi e^{-2\hat{\phi}} \hat{k}^2 \sqrt{|\det(\partial_i \hat{X}^\mu \partial_j \hat{X}^\nu \hat{\Pi}_{\mu\nu} - \hat{k}^{-2} \hat{F}_i \hat{F}_j)|} + \text{WZ}, \quad (38)$$

with

$$\hat{F}_i = \partial_i \hat{S} - \partial_i \hat{X}^\mu \hat{k}^\nu \hat{B}_{\mu\nu}. \quad (39)$$

A reduction of the KKh action over the  $z$ -direction gives

$$S_{\text{KKh}} = -T_{\text{KKh}} \int d^6 \xi e^{-2\phi} k \sqrt{|\det(\partial_i X^\mu \partial_j X^\nu g_{\mu\nu} - k^{-2} F'_i F'_j)|} + \text{WZ}. \quad (40)$$

Furthermore, we have

$$F'_i = \partial_i S - B_i. \quad (41)$$

Combining the above reductions we see that the P5h and KKh actions reduce to two actions in nine dimensions that differ by the following interchanges:

$$k \leftrightarrow k^{-1}, \quad A \leftrightarrow B, \quad (42)$$

which are exactly Buscher's rules in nine-dimensional language [28]. This proves the  $T$ -duality between the P5h and KKh actions.

#### 4. New Developments

After the conference new developments took place in which it became evident that gauged sigma models not only play a role in the description of the KK monopole but are also relevant to describe the dynamics of massive branes, i.e. branes that move in a background with a nonzero cosmological constant. These new developments are described in [29]. The first observation relevant to these developments is the following one. The 11-dimensional origin of the D-2-brane requires a worldvolume duality transformation of the Born-Infeld (BI) vector into a scalar [30]. This dualization proceeds in the standard way for massless backgrounds, i.e.  $m = 0$ , but is seemingly problematic for  $m \neq 0$  due to the presence of a topological mass term. It was noted in [22] that one can dualize *on-shell* and this dualization leads to the following line element for the eleventh scalar:

$$\partial_i X^{11} - m V_i. \quad (43)$$

In other words, the general line element is given by

$$\partial_i X^\mu - V_i k^\mu, \quad (44)$$

with  $k^\mu = m \delta^{\mu 11}$ . But this is exactly the line element of a gauged sigma model considered in the context of the KK monopole:

$$\partial_i X^\mu - C_i k^\mu. \quad (45)$$

The suggestion made was that gauged sigma models should also have a role to play in the description of massive branes.

Soon after the conference a paper appeared [31] where it was shown that the above-mentioned duality procedure could even be done *off-shell* by introducing an auxiliary 1-form and an explicit form of the massive M-2-brane action was given. It was suggested that the auxiliary 1-form played the role of the auxiliary gauge field in a gauged sigma model. The precise identification of the massive M-2-brane action as a gauged sigma model was subsequently made in [32].

It has turned out that the relation between the massive M-2-brane and gauged sigma models also applies to the other branes of M-theory, in particular the M-0-brane and M-5-brane [29]. We briefly summarize the different cases below (for more details, see [29]).

##### 4.1. The massive M-0-brane

We first consider the action of the massless M-0-brane:

$$\hat{S}[\hat{X}^\mu, \gamma] = -\frac{p}{2} \int d\xi \sqrt{|\gamma|} \gamma^{-1} \partial_\xi \hat{X}^\mu \partial_\xi \hat{X}^\nu \hat{g}_{\mu\nu}, \quad (46)$$

where  $p$  is a constant with the dimensions of mass. This action is known to give upon direct dimensional reduction the action of the D-0-brane of Type IIA superstring theory (see e.g. [33]). Our goal is to obtain an effective action with 11-dimensional target space from which one can derive the effective action of the massive D-0-brane. Our construction requires that the metric has an isometry generated by a Killing vector  $\hat{k}^\mu$ . To obtain a gauged sigma model one simply replaces the derivative to  $\xi$  by the covariant derivative

$$D_\xi \hat{X}^\mu = \partial_\xi \hat{X}^\mu - \frac{m}{2} (2\pi\alpha') b_\xi \hat{k}^\mu, \quad (47)$$

where  $b_\xi$  is an auxiliary gauge field. We thus obtain the following action:

$$\hat{S}_{\text{gauged}}[\hat{X}^\mu, b_\xi, \gamma] = \quad (48)$$

$$-\frac{p}{2} \int d\xi \sqrt{|\gamma|} \gamma^{-1} D_\xi \hat{X}^\mu D_\xi \hat{X}^\nu \hat{g}_{\mu\nu}.$$

Now we want to perform the dimensional reduction of the gauged action in the direction associated to the isometry. As a first step, using a coordinate system adapted to the isometry



( $\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}\nu}$ ) we rewrite the background fields in 10-dimensional form, obtaining

$$\hat{S} [X^\mu, c^{(0)}, b_\xi, \gamma,] = -\frac{p}{2} \int d\xi \sqrt{|\gamma|} \gamma^{-1} \left[ e^{-\frac{2}{3}\phi} g_{\xi\xi} - (2\pi\alpha')^2 e^{\frac{4}{3}\phi} \mathcal{G}^{(1)\xi}_\xi \right], \quad (49)$$

with

$$\begin{cases} g_{\xi\xi} &= \partial_\xi X^\mu \partial_\xi X^\nu g_{\mu\nu}, \\ C^{(1)}_\xi &= \partial_\xi X^\mu C^{(1)}_\mu, \\ \mathcal{G}^{(1)}_\xi &= \partial_\xi c^{(0)} + \frac{1}{2\pi\alpha'} C^{(1)}_\xi - \frac{m}{2} b_\xi. \end{cases} \quad (50)$$

$\mathcal{G}^{(1)}_\xi$  is the gauge-invariant “field” strength of  $c^{(0)}$ . The worldvolume 0-form  $c^{(0)}$  is related to the original 11-dimensional coordinate  $Y$  by

$$Y = (2\pi\alpha') c^{(0)}. \quad (51)$$

Next, we eliminate  $c^{(0)}$  (or, equivalently,  $Y$ ) by using its equation of motion which essentially says that the momentum of the particle in the direction  $Y$  is constant. Using standard techniques this leads to the action of a massive D-0-brane:

$$\begin{aligned} \tilde{S}' [X^\mu, b_\xi] &= -|P_y| \int d\xi e^{-\phi} \sqrt{|g_{\xi\xi}|} \\ &+ P_y \int d\xi \left( C^{(1)}_\xi - \frac{m}{2} (2\pi\alpha') b_\xi \right), \end{aligned} \quad (52)$$

with  $b_\xi$  being the Born-Infeld vector “field”.

## 4.2. The massive M-2-brane

Starting from the massless M-2-brane action we perform a similar gauging as in the particle case. At the same time, however, we want the resulting gauged action to remain invariant under the gauge transformations of the 3-form  $\hat{C}$ . A straightforward Noether procedure leads to the following action [32]

$$\begin{aligned} \hat{S}_{\text{gauged}} [\hat{X}^{\hat{\mu}}, \hat{b}_i] &= \\ &-T_{M2} \int d^3\hat{\xi} \sqrt{|D_i \hat{X}^{\hat{\mu}} D_j \hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}}|} \\ &+ \frac{T_{M2}}{3!} \int d^3\hat{\xi} \hat{\epsilon}^{ijk} \left\{ D_i \hat{X}^{\hat{\mu}} D_j \hat{X}^{\hat{\nu}} D_k \hat{X}^{\hat{\rho}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} \right. \\ &\left. - 3 \frac{m}{2} (2\pi\alpha')^2 \hat{b}_i \partial_j \hat{b}_k \right\}, \end{aligned} \quad (53)$$

where

$$D_i \hat{X}^{\hat{\mu}} = \partial_i \hat{X}^{\hat{\mu}} - \frac{m}{2} (2\pi\alpha') \hat{b}_i(\hat{\xi}) \hat{k}^{\hat{\mu}}(\hat{X}). \quad (54)$$

Below we describe the direct and double dimensional reduction of this action.

### 4.2.1. Direct Dimensional Reduction: the Massive D-2-Brane

By assumption the background has an isometry and we can perform direct dimensional reduction of the action Eq. (53) in the direction associated to the isometry that we have gauged. This amounts to a simple rewriting of the background fields from 11-dimensional to 10-dimensional form. We thus find the action of [31]:

$$\begin{aligned} \tilde{S} [X^\mu, \hat{c}^{(0)}, \hat{b}_i] &= \\ &-T_{M2} \int d^3\hat{\xi} e^{-\phi} \sqrt{|g_{ij} - (2\pi\alpha')^2 e^{2\phi} \mathcal{G}^{(1)}_i \mathcal{G}^{(1)}_j|} \\ &+ \frac{T_{M2}}{3!} \int d^3\hat{\xi} \hat{\epsilon}^{ijk} \left\{ C^{(3)}_{ijk} - 3C^{(1)}_i B_{jk} \right. \\ &\left. + 3(2\pi\alpha') \mathcal{G}^{(1)}_i B_{jk} - 3 \frac{m}{2} (2\pi\alpha')^2 \hat{b}_i \partial_j \hat{b}_k \right\}, \end{aligned} \quad (55)$$

where

$$\begin{cases} g_{ij} &= \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}, \quad \text{etc.} \\ \mathcal{G}^{(1)}_i &= \partial_i \hat{c}^{(0)} + \frac{1}{2\pi\alpha'} C^{(1)}_i - \frac{m}{2} \hat{b}_i. \end{cases} \quad (56)$$

The worldvolume 0-form  $\hat{c}^{(0)}$  is related to the original 11-dimensional coordinate  $Y$  as in eq. (51) and transforms as follows:

$$\delta \hat{c}^{(0)} = -\frac{1}{2\pi\alpha'} \Lambda^{(0)} + \frac{m}{2} \hat{\rho}^{(0)}. \quad (57)$$

The equivalence between this action and the usual effective action for the massive D-2-brane which only contains the BI vector field (and not the worldvolume scalar  $\hat{c}^{(0)}$ ) can be seen by dualizing  $\hat{c}^{(0)}$  into a 1-form  $\hat{a}_i$ . The equation of motion of  $\hat{b}_i$  then reads

$$\hat{a}_i = \hat{b}_i \quad (58)$$

and the action reduces to that of a massive D-2-brane:

$$\begin{aligned}
 S[X^\mu, \hat{b}_i] = & \\
 & -T_{M2} \int d^3\xi \, e^{-\phi} \sqrt{|g_{ij} + 2\pi\alpha' \hat{\mathcal{F}}_{ij}|} \\
 & + \frac{T_{M2}}{3!} \int d^3\xi \, \epsilon^{ijk} \left\{ C^{(3)}_{ijk} - 3(2\pi\alpha') C^{(1)}_i \hat{\mathcal{F}}_{jk} \right. \\
 & \left. + 3 \frac{m}{2} (2\pi\alpha')^2 \hat{b}_i \partial_j \hat{b}_k \right\}. \quad (59)
 \end{aligned}$$

where the worldvolume 1-form field  $\hat{b}_i$  has become the Born-Infeld (BI) vector field.

#### 4.2.2. Double Dimensional Reduction: the type IIA String

We now consider the double dimensional reduction of the massive M-2-brane worldvolume effective action. We will see that this reduction leads to the usual (“massless”) type IIA string action.

It is convenient to perform the double dimensional reduction of Eq. (53) in two steps. First, we perform a direct dimensional reduction, obtaining Eq. (55). In a second step we eliminate the target space coordinate  $Y$  together with the worldvolume coordinate  $\xi^2$ . This is done by setting

$$\hat{c}^{(0)} = \frac{1}{2\pi\alpha'} \xi^2, \quad \partial_2 X^\mu = \partial_2 \hat{b}_i = 0. \quad (60)$$

The auxiliary worldvolume 1-form  $\hat{b}_i$  is reduced as follows:

$$\hat{b}_i = b_i, \quad \hat{b}_2 = s. \quad (61)$$

Substitution of the above Ansatz in the action leads to

$$\begin{aligned}
 \tilde{S}[X^\mu, b_i, s] = & \\
 & -T_{M2} l \int d^2\xi \, \left(1 - \frac{m}{2} (2\pi\alpha') s\right) \sqrt{|g_{ij}|} \\
 & + \frac{T_{M2} l}{2} \int d^2\xi \, \epsilon^{ij} \left\{ \left(1 - \frac{m}{2} (2\pi\alpha') s\right) B_{ij} \right. \\
 & \left. - m (2\pi\alpha')^2 s \partial_i b_j \right\}, \quad (62)
 \end{aligned}$$

where we have taken  $\xi^2 \in [0, l]$ . We can now safely eliminate the worldvolume vector field by

using its equations of motion<sup>8</sup>. First, the equation of motion for the scalar  $s$  is

$$\epsilon^{ij} \mathcal{F}_{ij} = (2\pi\alpha') \sqrt{|g_{ij}|}, \quad (63)$$

and substituting it into the above action both  $s$  and  $b_i$  are eliminated and the resulting action is that of the type IIA string in the Nambu-Goto form:

$$\begin{aligned}
 S[X^\mu] = & -T_{M2} l \int d^2\xi \, \sqrt{|g_{ij}|} \\
 & + \frac{T_{M2} l}{2} \int d^2\xi \, \epsilon^{ij} B_{ij}. \quad (64)
 \end{aligned}$$

#### 4.3. The massive M-5-brane

The massive M-5-brane can be treated at the same footing as the M-0-brane and M-2-brane discussed above. Since the formulae involved are rather complicated we will not give the details here (they can be found in [29]). However, an additional subtlety occurs which is most easily described by first considering the 10-dimensional massive p-5A-brane which is nothing but the direct dimensional reduction of the massive M-5-brane.

The point is that the construction of the WZ term in the worldvolume action of the massive p-5A-brane requires a dualization of the massive NS/NS target space 2-form field along the lines recently discussed in [34]. A noteworthy feature is that, whereas in the usual formulation of IIA supergravity the R-R 1-form  $C^{(1)}$  is a Stueckelberg field that gets “eaten up” by the NS/NS 2-form  $B$  which becomes massive:

$$\begin{cases} C^{(1)} & \rightarrow \text{Stueckelberg field,} \\ B & \rightarrow \text{massive field,} \end{cases}$$

in the dual formulation the situation is reversed: the dual NS/NS 6-form  $\tilde{B}_{\text{IIA}}$  becomes a Stueckelberg field giving mass to the dual R-R 7-form  $C^{(7)}$ :

$$\begin{cases} C^{(7)} & \rightarrow \text{massive field,} \\ \tilde{B}_{\text{IIA}} & \rightarrow \text{Stueckelberg field.} \end{cases}$$

<sup>8</sup>We ignore here the fermionic terms which in the supersymmetric case are to be added to the field equations. It might well be that, in order to construct a kappa-symmetric massive superstring action, it is more convenient to use the form of the action given in (62).

An immediate consequence of the above observation is that, since  $\tilde{B}_{\text{IIA}}$  occurs as the leading term of the WZ term of the p-5A-brane, we must introduce an independent auxiliary 6-form worldvolume field in order to cancel the Stueckelberg transformations of  $\tilde{B}_{\text{IIA}}$ . This is on top of an auxiliary worldvolume 1-form which must be introduced in order to construct a WZ term that is invariant under the “massive gauge transformations” of massive IIA supergravity. The massive p-5A-brane therefore contains extra couplings to a worldvolume 1-form and 6-form that are absent in the massless case. The situation is similar to that of a D-0-brane in a massive background. In that case the Stueckelberg variation of the leading term  $C^{(1)}$  in the WZ term is cancelled by an auxiliary BI 1-form field that couples to the D-0-brane with a strength proportional to the mass parameter  $m$ .

## 5. Conclusions

We have reviewed different aspects of branes in string theory and M-theory. In particular, we have argued that the worldvolume theory describing the dynamics of a Kaluza-Klein monopole is given by a gauged sigma model. We have also briefly reviewed some recent applications of gauged sigma models to the description of massive branes, i.e. branes moving in a background with a nonzero cosmological constant.

One interesting outcome of the construction of the massive IIA 5-brane is that it contains a coupling to a worldvolume 6-form  $c^{(6)}$  with the strength of the coupling proportional to  $m$ :

$$S_{\text{massive 5-brane}} \sim \int d^6\xi \, m \epsilon^{i_1 \dots i_6} c_{i_1 \dots i_6}. \quad (65)$$

A similar coupling occurs in the massive D-0-brane action:

$$S_{\text{massive D0-brane}} \sim \int d\xi \, m b_\xi. \quad (66)$$

For the massive D0-brane, this new coupling, which is absent in the massless case, is known to have implications for the anomalous creation of branes [35,36]. It can be argued that the new

couplings we find in the case of the solitonic 5-brane have similar implications [29].

It would be interesting to see whether the gauged sigma model approach can also be applied to describe a massive KK11 monopole and/or the conjectured M9-brane. Concerning the KK11-monopole, the situation is unclear at the time of writing. Note that a new feature in this case is that the gauged sigma model is already needed to describe the dynamics of the KK11-monopole in a massless background.

Finally, one might wonder whether our results shed new light on the evasive 11-dimensional 9-brane (see also [20]). A standard argument against the 9-brane is that the corresponding 10-dimensional worldvolume field theory does not allow multiplets containing a single scalar to indicate the position of the 9-brane. A way out of this is to assume that the 9-brane is really an 8-brane with an extra isometry in one of the 2 transverse directions, leading to a gauged sigma-model. Now, we are dealing with a nine-dimensional field theory which naturally contains a vector multiplet with a single scalar. It would be interesting to pursue this line of thought further and see whether it leads to a proper formulation of the long sought for 11-dimensional 9-brane.

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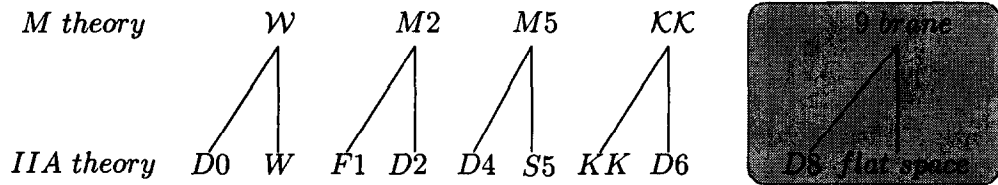


Figure 1. **The relation between  $d = 10$  IIA and  $d = 11$  solutions:** Vertical lines imply direct dimensional reduction, diagonal lines double dimensional reduction. The shadowed area indicates the relationship between known ten-dimensional solutions and a conjectured 9-brane in  $d = 11$ .